Test

- Derivatives
  - inverse trig functions
  - exponential / logarithmic
- Implicit Differentiation
- Linearization
- Inflection pts.
- Solve log problem

$\boxed{\text{HW}}$

study for test

practice problems and solutions on the following pages

① $\frac{d}{dx}\left[\arcsin(5x)\right]$

② $\frac{d}{dx}\left[\arctan e^{x}\right]$

③ $\frac{d}{dx}\left[\operatorname{arcsec}\left(\frac{x}{2}\right)\right], \quad x>0$

④ $\frac{d}{dx}\left[\arcsin x + x\sqrt{1-x^2}\right]$

⑤ $\frac{d}{dx}\left[\ln(1+e^{2x})\right]$

⑥ $\frac{d}{dx}\left[e^{-x}\ln x\right]$

7. Find $\frac{dy}{dx}$ of $xe^y - 10x + 3y = 0$

8. Find $\frac{dy}{dx}$ of $e^{xy} + x^2 - y^2 = 10$

9. $\frac{d}{dx}\left[\log_3 \frac{x\sqrt{x-1}}{2}\right]$

10. $\frac{d}{dx}\left[t^2 2^t\right]$

11. $\frac{d}{dx}\left[\arctan\sqrt{x}\right]$

12. $\frac{d}{dx}\left[\frac{\arcsin 3x}{x}\right]$

Practice Problems for Transcendentals

2/9/11

① $\frac{d}{dx}\left[\arcsin(5x)\right] = \frac{1}{\sqrt{1-(5x)^2}} \cdot 5 = \boxed{\frac{5}{\sqrt{1-25x^2}}}$

② $\frac{d}{dx}\left[\arctan(e^x)\right] = \frac{1}{1+(e^x)^2} \cdot e^x = \boxed{\frac{e^x}{1+e^{2x}}}$

③ $\frac{d}{dx}\left[\operatorname{arcsec}\left(\frac{x}{2}\right)\right] = \frac{1}{\left|\frac{x}{2}\right|\sqrt{\left(\frac{x}{2}\right)^2-1}} \cdot \frac{1}{2} = \boxed{\frac{1}{x\sqrt{\frac{x^2}{4}-1}} \text{ or } \frac{2}{x\sqrt{x^2-4}}}$

$x > 0$

④ $\frac{d}{dx}\left[\arcsin x + x\sqrt{1-x^2}\right] = \frac{1}{\sqrt{1-x^2}} + x \cdot \frac{1}{2}\frac{1}{\sqrt{1-x^2}} \cdot -2x + \sqrt{1-x^2}$

$= \frac{1}{\sqrt{1-x^2}} + \frac{-2x^2}{2\sqrt{1-x^2}} + \frac{\sqrt{1-x^2}}{1}$ common denominator

$= \frac{2 - 2x^2 + 2 - 2x^2}{2\sqrt{1-x^2}} = \frac{4-4x^2}{2\sqrt{1-x^2}}$

$\boxed{= \frac{2-2x^2}{\sqrt{1-x^2}}}$

$\sqrt{1-x^2}$

(5) $\frac{d}{dx}\left[\ln\left(1+e^{2x}\right)\right] = \frac{1}{1+e^{2x}} \cdot e^{2x} \cdot 2 = \boxed{\frac{2e^{2x}}{1+e^{2x}}}$

(6) $\frac{d}{dx}\left[e^{-x}\ln x\right] = e^{-x} \cdot \frac{1}{x} + \ln x \cdot -e^{-x} = \boxed{\frac{e^{-x}}{x} + -\ln(x)\, e^{-x} = \frac{e^{-x} - xe^{-x}\ln x}{x}}$

7. $xe^y - 10x + 3y = 0 \rightarrow xe^y\frac{dy}{dx} + e^y - 10 + 3\frac{dy}{dx} = 0$

$$\frac{dy}{dx}(xe^y + 3) = 10 - e^y \rightarrow \boxed{\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}}$$

8. $e^{xy} + x^2 - y^2 = 10 \rightarrow e^{xy}\left(x\frac{dy}{dx} + y\right) + 2x - 2y\frac{dy}{dx} = 0$

$$e^{xy}x\frac{dy}{dx} + e^{xy}y + 2x - 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}(e^{xy}x - 2y) = -2x - e^{xy}y$$

$$\boxed{\frac{dy}{dx} = \frac{-2x - e^{xy}y}{e^{xy}x - 2y}}$$

9. $\frac{\partial}{\partial x}\left[\log_3 \frac{x\sqrt{x-1}}{2}\right] = \frac{1}{\frac{x\sqrt{x-1}}{2}\ln 3}\cdot\left(\cancel{\tfrac{1}{2}}x\cdot\frac{1}{2\sqrt{x-1}} + \sqrt{x-1}\right)$

$$\frac{\cancel{2}\,1}{x\sqrt{x-1}\ln 3}\cdot\left(\frac{x + 2x - 2}{2\sqrt{x-1}}\right)$$

$$\boxed{\frac{\cancel{3}x - \cancel{2}}{2x\ln(3)(x-1)} = \frac{\cancel{3}x - \cancel{2}}{\ln 3\,(2x^2 - 2x)}}$$

(10) $\frac{d}{dx}\left[t^2 2^t\right] = t^2 \cdot 2^t \ln(2) + 2^t \cdot 2t = \boxed{2^t\left(\ln 2\, t^2 + 2t\right)}$

(11) $\frac{d}{dx}\left[\arctan\sqrt{x}\right] = \frac{1}{(\sqrt{x})^2+1} \cdot \frac{1}{2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}(x+1)}}$

(12) $\frac{d}{dx}\left[\frac{\arcsin(3x)}{x}\right] = \frac{x \cdot \frac{1}{\sqrt{1-(3x)^2}} \cdot 3 - \sin^{-1}(3x)}{x^2}$

$= \frac{\frac{3x}{\sqrt{1-9x^2}} - \sin^{-1}(3x)}{x^2} = \boxed{\frac{3x - \sqrt{1-9x^2}\,\sin^{-1}(3x)}{x^2\sqrt{1-9x^2}}}$